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Bubbles: a technique to reveal the use of information in recognition tasks

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Abstract

Everyday, people flexibly perform different categorizations of common faces, objects and scenes. Intuition and scattered evidence suggest that these categorizations require the use of different visual information from the input. However, there is no unifying method, based on the categorization performance of subjects, that can isolate the information used. To this end, we developed Bubbles, a general technique that can assign the credit of human categorization performance to specific visual information. To illustrate the technique, we applied Bubbles on three categorization tasks (gender, expressive or not and identity) on the same set of faces, with human and ideal observers to compare the features they used.

Keywords: Bubbles; Recognition tasks; Categorizations

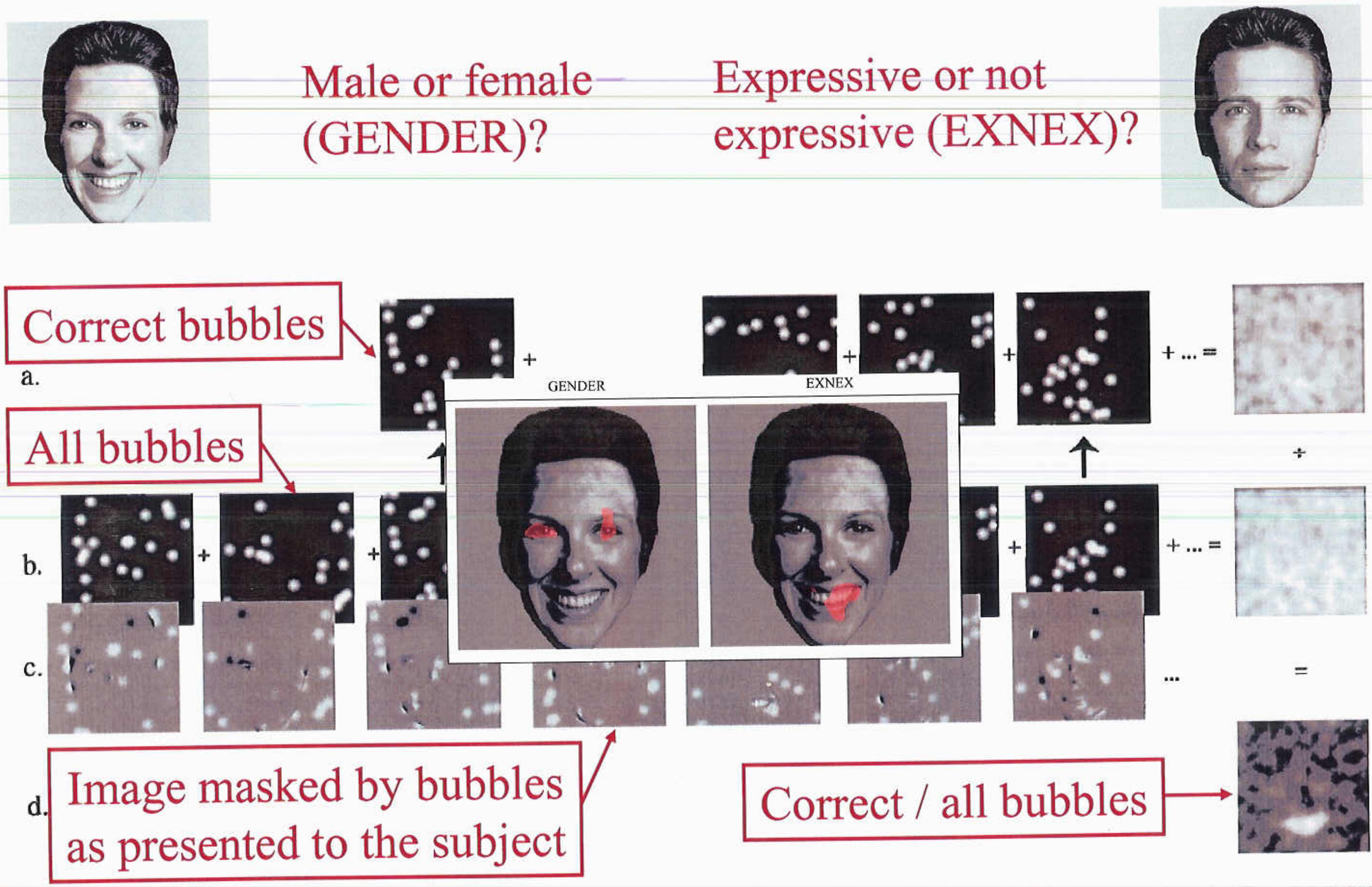
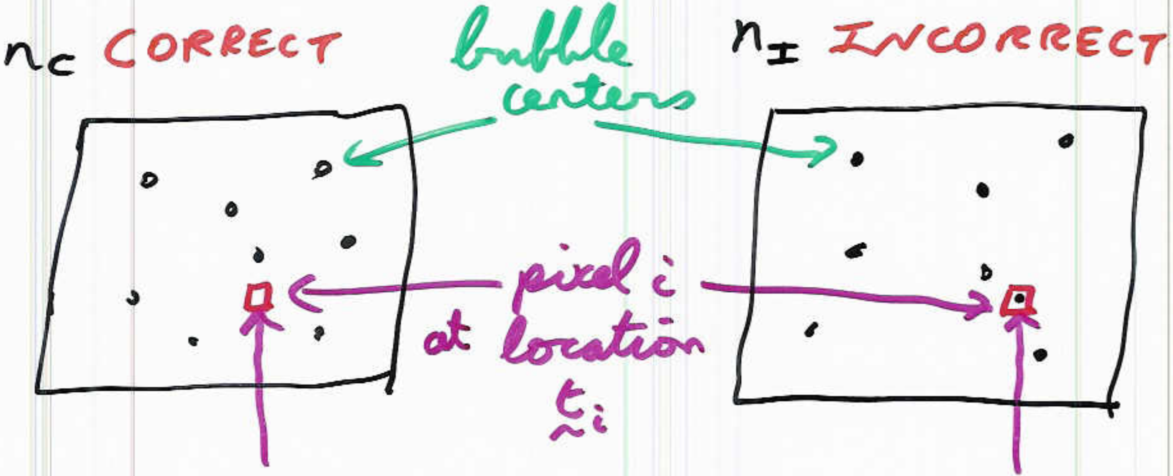


Fig. 3. This figure illustrates Bubbles in experiment 1 for the EXNEX task. In (a), the bubbles leading to a correct categorization are added together to form the CorrectPlane (the rightmost greyscale picture). In (b), all bubbles (those leading to a correct and incorrect categorizations) are added to form TotalPlane (the rightmost greyscale picture). In (c), examples of experimental stimuli as revealed by the bubbles of (b). It is illustrative to judge whether each sparse stimulus is expressive or not. ProportionPlane (d) is the division of CorrectPlane with TotalPlane. Note the whiter mouth area (the greyscale has been renormalized to facilitate interpretation). See Fig. 2 for the outcome of experiment 1.

Bubbles analysis: usual 2-sample test for proportions:



$$P_C = \frac{\#\text{ bub centres}}{n_C} \qquad P_I = \frac{\#\text{ bub centres}}{n_I}$$

$$P = \frac{n_C P_C + n_I P_I}{n_C + n_I}$$

$$X(\underset{\sim}{t}_i) = \left(\frac{P_C - P_I}{\sqrt{P(1-P)\left(\frac{1}{n_C} + \frac{1}{n_I}\right)}} \right) \text{ smooth}$$

$\sim N(0,1)$ under H_0

$$P\left(\max_{\underset{\sim}{t} \in S} X(\underset{\sim}{t}) \geq x \right) = \,?$$

Problem: Detect a localised signal in $X(\underset{\sim}{t})$ inside a set S (= brain)

Test statistic: $\max_{\underset{\sim}{t} \in S} X(\underset{\sim}{t})$ (likelihood ratio test under certain models for signal)

P-value? $P\left(\max_{\underset{\sim}{t} \in S} X(\underset{\sim}{t}) \geq x\right)$

$= P\left(X(\underset{\sim}{t}) \geq x \text{ for some } \underset{\sim}{t} \in S\right)$

Bonferroni? $\leq$ (# voxels) $P\left(X(\underset{\sim}{t}) \geq x\right)$

– too conservative!

For large x; A_x sparse,

$$\left(\max_{\underset{\sim}{t} \in S} X(\underset{\sim}{t}) \geq x\right) \approx EC(S \cap A_x)$$

indicator: $\begin{cases} 1 & \text{if true} \\ 0 & \text{if false} \end{cases}$

Take expectations:

$$P\left(\max_{\underset{\sim}{t} \in S} X(\underset{\sim}{t}) \geq x\right) \approx E(EC(S \cap A_x))$$

Asymptotic results: Rice (1945), Leadbetter, Lindgren, Rootzén, Belyaev, Piterbarg (1970's)

BUT

Exact results for all x, which are much more accurate for P-value.

Abstract tubes (Naiman, Wynn, 1992)

Search region $S = \{\boldsymbol{t}_1, \boldsymbol{t}_2, \ldots, \boldsymbol{t}_n\}$ is discrete

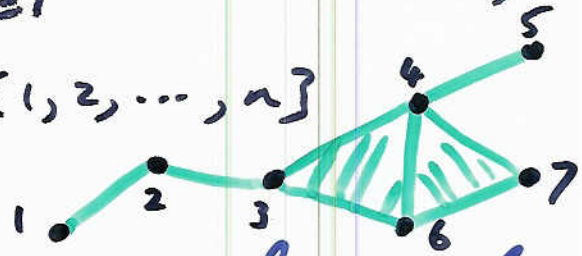
$A_i = \{X(\boldsymbol{t}_i) \geq x\}$, $P(\max_i X(\boldsymbol{t}_i) \geq x) = P\left(\bigcup_{i=1}^{n} A_i\right)$

$$\sum_i P(A_i) - \sum_i \sum_j P(A_i \cap A_j) \leq \boxed{P\left(\bigcup_i A_i\right) \leq \sum_i P(A_i)}$$

Bonferroni

"improved" Bonferroni: $-\sum_{i=1}^{n-1} P(A_i \cap A_{i+1})$

Simplicial complex $\subset 2^{\{1,2,\ldots,n\}}$



Sub-simplicial complex is a subset of simplices that can occur (with prob >0)

(identifying $\{i,j\}$ "edge" with $A_i \cap A_j$ etc.
"triangle" $\{i,j,k\}$ with $A_i \cap A_j \cap A_k$)

EC of a sub simplicial complex is

$$EC = \#\text{points} - \#\text{edges} + \#\text{triangles} - \ldots$$

Abstract tube T is simplicial complex s.t.

$$EC(\text{sub simplicial complex}) = 1$$

Proof:

$$P(A_1 \cup A_2 \cup A_3 \cup \cdots \cup A_n)$$

$$= P\left(A_1 \cup (A_2 \setminus A_1) \cup (A_3 \setminus A_2) \cup \cdots \cup (A_n \setminus A_{n-1})\right)$$

BONFERRONI:

$$\leq P(A_1) + P(A_2 \setminus A_1) + P(A_3 \setminus A_2) + \cdots + P(A_n \setminus A_{n-1})$$

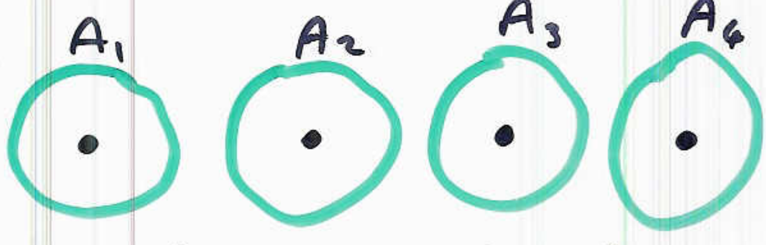
$$= P(A_1) + P(A_2) - P(A_1 \cap A_2) + P(A_3) - P(A_2 \cap A_3) + \cdots$$

Theorem If T is an abstract tube then

$$P\Big(\bigcup_i A_i\Big) = \sum_{\text{points} \in T} P(A_i) - \sum_{\text{edges} \in T} P(A_i \cap A_j)$$

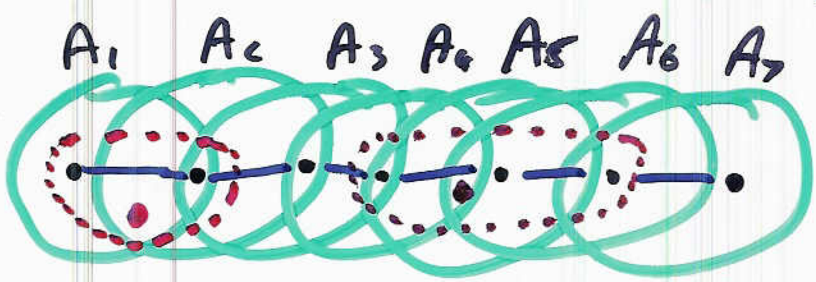
$$+ \sum_{\text{triangles} \in T} P(A_i \cap A_j \cap A_k) - \sum_{\text{tetrahedra} \in T} \cdots$$

e.g.



$P(\bigcup A_i) = \sum P(A_i)$

i.e. Bonferroni is exact



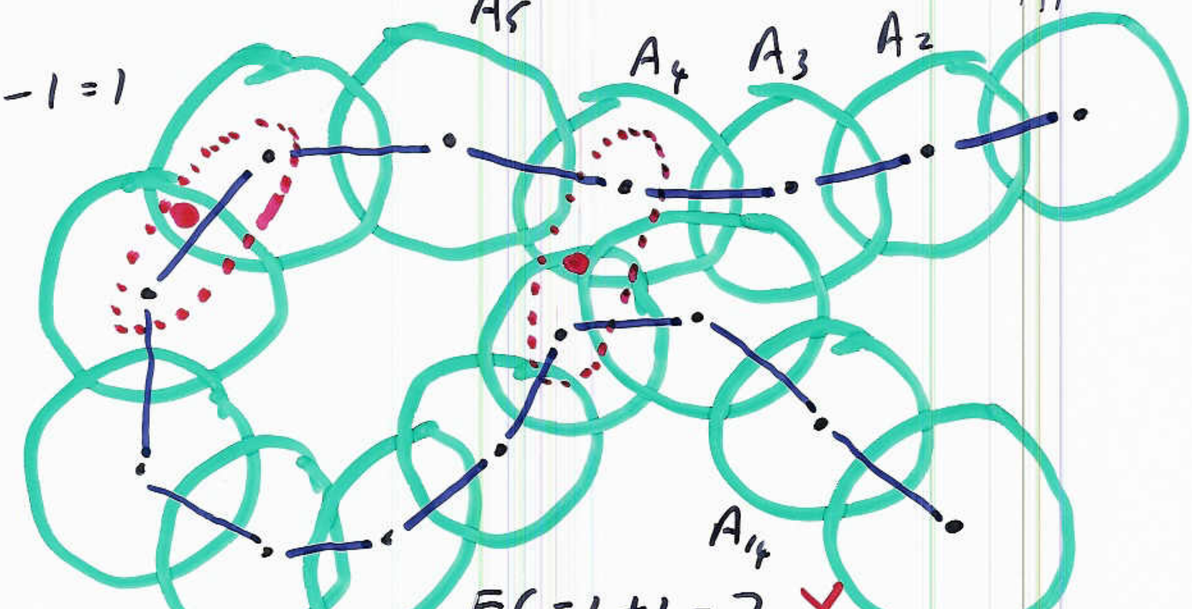
$$P\Big(\bigcup A_i\Big) = \sum P(A_i) - \sum P(A_i \cap A_{i+1})$$

i.e. improved Bon is exact.

$EC = 2 - 1 = 1$ ✓

$EC = 3 - 2 = 1$ ✓

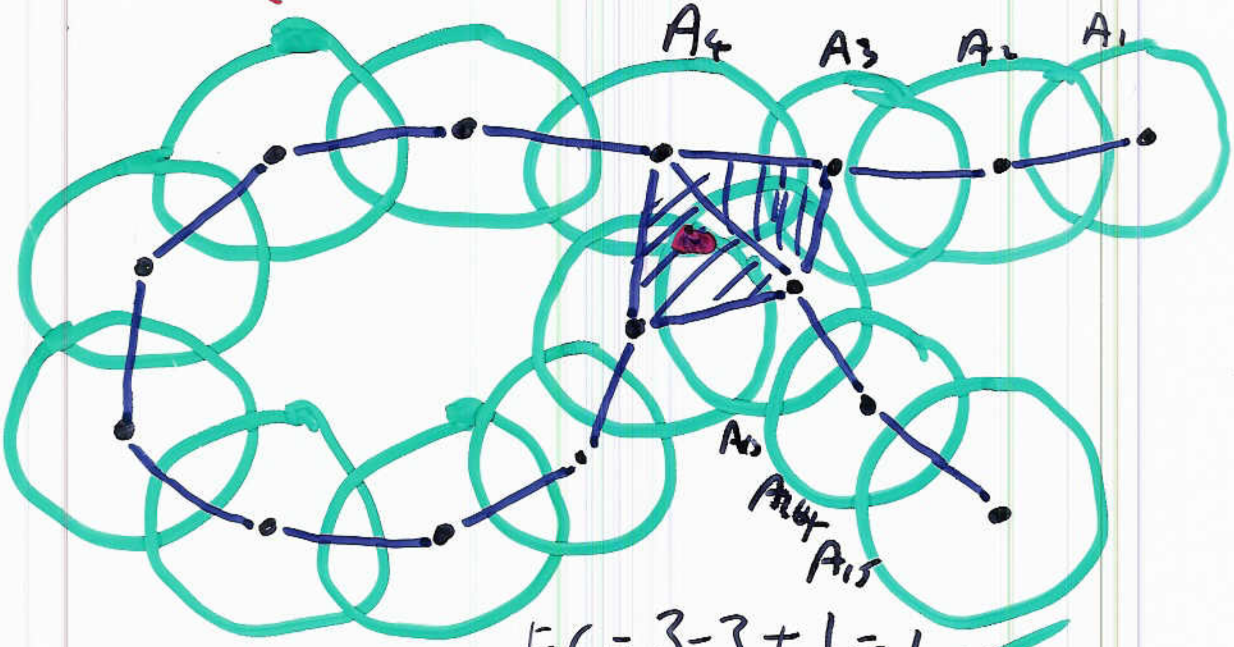
$EC = 1 + 1 - 1 = 1$ ✓



$EC = 1 + 1 = 2$ ✗ A_{15}

not an abstract tube

$$P(\cup A_i) \neq \sum P(A_i) - \sum P(A_i \cap A_{i+1})$$



$EC = 3 - 3 + 1 = 1$ ✓

is an abstract tube

$$P(\cup A_i) = \sum P(A_i) - \sum P(A_i \cap A_{i+1})$$
$$- P(A_3 \cap A_{13}) - P(A_4 \cap A_{13}) - P(A_4 \cap A_{12})$$
$$+ P(A_3 \cap A_4 \cap A_{13}) + P(A_4 \cap A_{12} \cap A_{13}).$$

Proof "Trivial":

A sub simp. comp. = $\{A_i\}$ that can occur

= "excursion set"

= $\{ i : X(\xi_i) \geq x \}$.

Condition for an abstract tube is:

EC("excursion set") = 1 whenever something occurs (non-empty)

$$\Rightarrow E(EC) = P(\text{something occurs}) = P(\cup A_i)$$

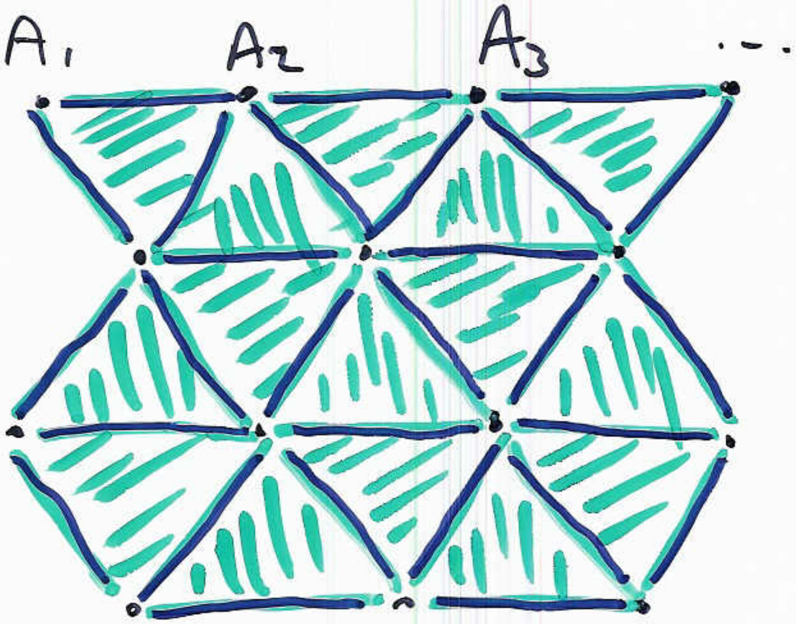
THEOREM

BUT:

$$E(EC) = E(\#\text{points} - \#\text{edges} + \#\text{faces} - \cdots)$$

$$= \sum_{\text{points}} P(A_i) - \sum_{\text{edges}} P(A_i \cap A_j) + \sum_{\text{faces}} P(A_i \cap A_j \cap A_k) \cdots\cdots$$

For a random field, the natural choice of simplicial complex is a triangular mesh in 2D:



which is almost an abstract tube if x is large,* in which case

$$P\left(\max_{\underset{\sim}{t} \in S} X(\underset{\sim}{t})\right) \approx E\left(EC(A_x \cap S)\right)$$

* $EC(\text{excursion set}) \approx \begin{cases} 1 & \text{if not empty} \\ 0 & \text{if empty} \end{cases}$

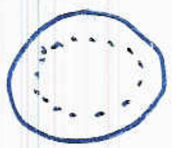
EULER CHARACTERISTIC

In 3D, $EC = \#\text{blobs} - \#\left\{\begin{array}{l}\text{handles}\\ \text{tunnels}\end{array}\right. + \#\text{hollows}$

e.g. $EC(\text{golf ball}) = 1$

$EC($doughnut  $) = 0$

$EC($pretzel  $) = -2$

$EC($tennis ball  $) = +2$

FORMAL DEFINITION:

$EC(\emptyset) = 0$, $EC\left(\begin{array}{c}\text{simply connected set,}\\ \text{homeomorphic to ball}\end{array}\right) = 1$

$EC(A \cup B) = EC(A) + EC(B) - EC(A \cap B)$.

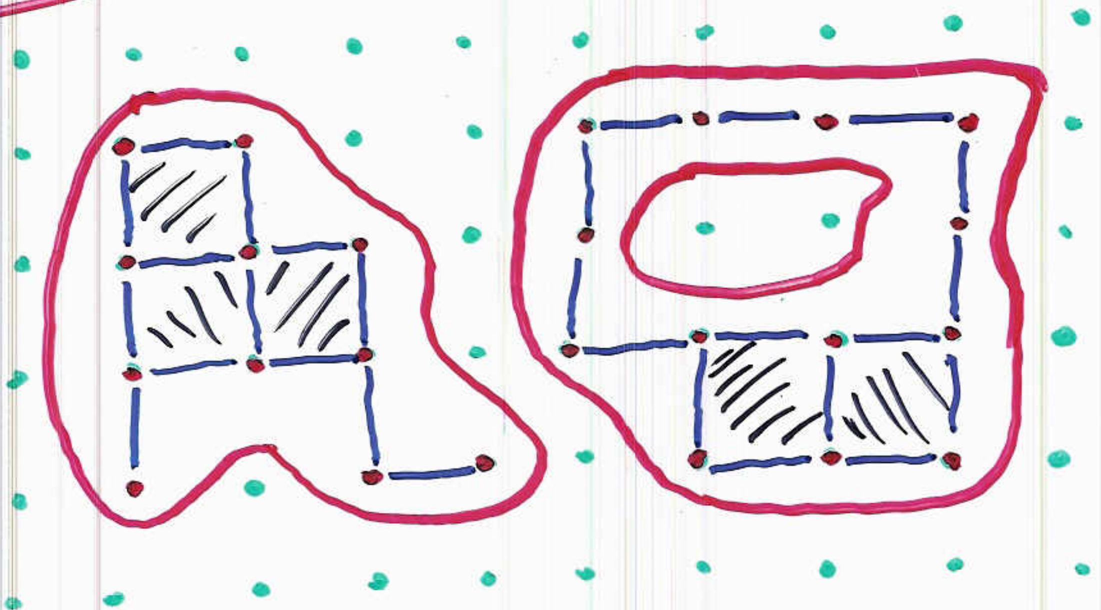
TWO APPLICATIONS:

1. Descriptive measure of "topology"
2. Estimate number of local "signals".

How to calculate EC

Cover set with a fine lattice:

eg. D=2:



EC = #points − #edges + #faces in set
$= 24 - 28 + 5 = 1$

D=3:

EC = #points − #edges + #faces − #cubes in set

etc.

<u>Model</u> the normalised galaxy density by a random field

$$X(\underset{\sim}{t}), \quad \underset{\sim}{t} \in \mathbb{R}^D$$

isotropic, smooth, $\sim N(0,1)$ at each point.

<u>Excursion set</u> $A_x = \{ \underset{\sim}{t} : X(\underset{\sim}{t}) \geq x \}$

<u>Theorem</u> (Adler, 1976, 1981):

$$\boxed{\rho_D(x) = E(EC(A_x)) = \frac{\lambda^{\frac{D}{2}}}{(2\pi)^{\frac{D+1}{2}}} He_{D-1}(x)\, e^{-\frac{1}{2}x^2}}$$

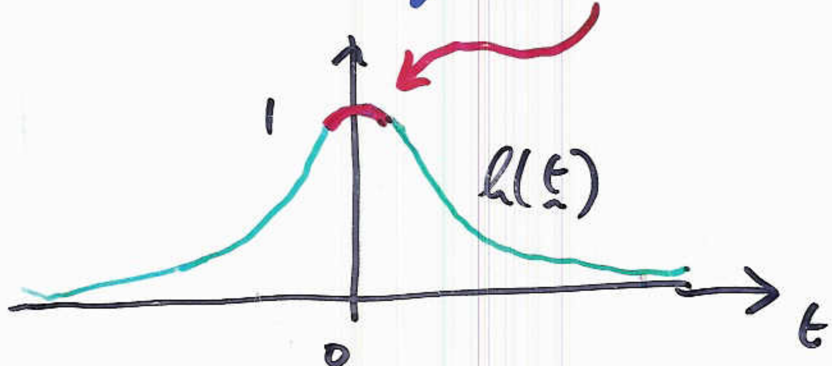
per unit volume

$$\lambda = \operatorname{Var}\left(\frac{\partial X}{\partial t_1}\right)$$

Hermite polynomial:

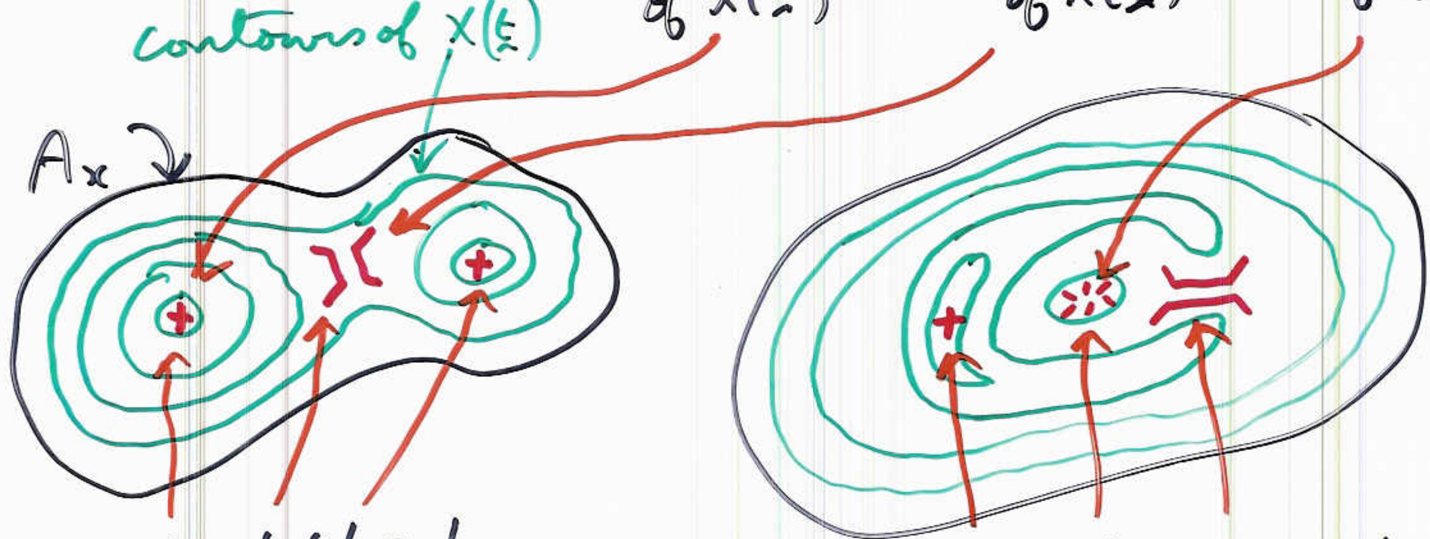
$D=3$: $He_2(x) = x^2 - 1$

<u>Note</u>: Depends on spatial correlation function $h(\underset{\sim}{t})$ of $X(\underset{\sim}{t})$ only through its curvature at the origin $\ddot{h}(0) = -\lambda \underset{D\times D}{I}$



Proof: Morse Theory (1969):

$D=2$: $EC(A_x) = \#$ maxima of $X(\underline{t})$ $-$ $\#$ saddles of $X(\underline{t})$ $+$ $\#$ minima of $X(\underline{t})$



$EC = +1 - 1 + 1 = 1$ $\qquad$ $EC = +1 + 1 - 1 = 1$

$$EC(A_x) = \sum_{\underline{t}} (X \geq x)(\dot{X} = 0)\, \mathrm{sign}(\det(-\ddot{X}))$$

$$E(EC(A_x)) = E\Big((X \geq x)\, \underbrace{\mathrm{sign}(\det(-\ddot{X}))\, \overbrace{|\det(-\ddot{X})|}^{\text{jacobian}}} \,\Big|\, \dot{X} = 0 \Big)\, P(\dot{X} = 0)$$

$$\boxed{\rho_0(x) = E\big((X \geq x)\det(-\ddot{X}) \,\big|\, \dot{X} = 0\big)\, P(\dot{X} = 0)}$$

(Brillinger, 1970)

$X, \dot{X}, \ddot{X} \sim$ jointly $N \longrightarrow$ result.

Recall: For large S,

EC density

$$E(EC(S \cap A_x)) \approx Vol(S)\, \rho_D(x)$$

Theorem (W, 1995)

$$\boxed{E(EC(S \cap A_x)) = \sum_{d=0}^{D} \mu_d(S)\, \rho_d(x)}$$

Intrinsic volumes

$$\mu_d(S) = \begin{cases} Vol(S) & \text{if } d = D \\ \displaystyle\int_{\partial S} \frac{\mathrm{detr}_{D-1-d}\left(\begin{matrix}\text{Curvature} \\ \text{matrix of } \partial S\end{matrix}\right)}{2\pi^{\frac{D-d}{2}} \big/ \Gamma\left(\frac{D-d}{2}\right)} & \text{if } d = 0, \ldots D-1. \end{cases}$$

$\mu_0(S) = EC(S)$ by Gauss-Bonnet theorem.

sum of determinants of all $D-1-d \times D-1-d$ principal minors of

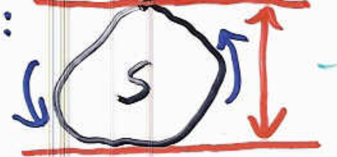
E.g. $D=3$: $$E(EC(S \cap A_x)) = Vol(S)\, \lambda^{\frac{3}{2}} \frac{(x^2-1)e^{-\frac{1}{2}x^2}}{(2\pi)^2}$$

$$+ \frac{1}{2}\,\text{Surface area}(S)\, \lambda \frac{x e^{-\frac{1}{2}x^2}}{(2\pi)^{\frac{3}{2}}}$$

$$+ 2\,\text{"Caliper diameter"}(S)\, \frac{\lambda^{\frac{1}{2}} e^{-\frac{1}{2}x^2}}{2\pi}$$

$$+ EC(S) \int_x^\infty \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2} dz$$

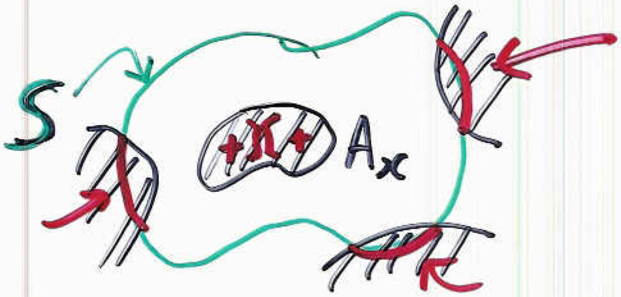
Mean diameter over all rotations of S:



Boundary correction terms

Proof:

① Count # max, saddles, min on the boundary of S as well as inside:



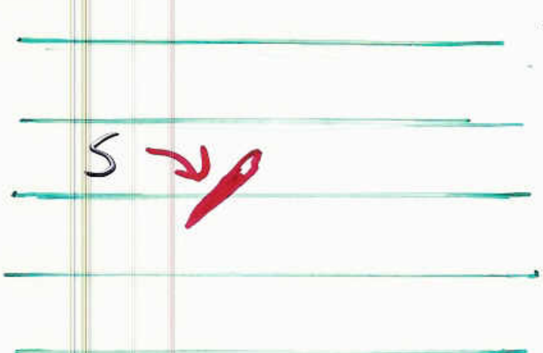
② "Kinematic Fundamental Formula" (Blaschke, 1935)

Fixed

$$E\big(EC(S \cap A_x)\big) = \sum_{d=0}^{D} \mu_d(S)\, \underbrace{\mu_{D-d}(A_x)\, a_d}_{\rho_d(x)}$$

Expectation over all rotations, translations of S

Generalisation of Buffon's needle (1757):



A $\quad E(EC(S \cap A)) =$ prob needle crosses cracks

$= \dfrac{2}{\pi}$

Replace Needle ← brain, S

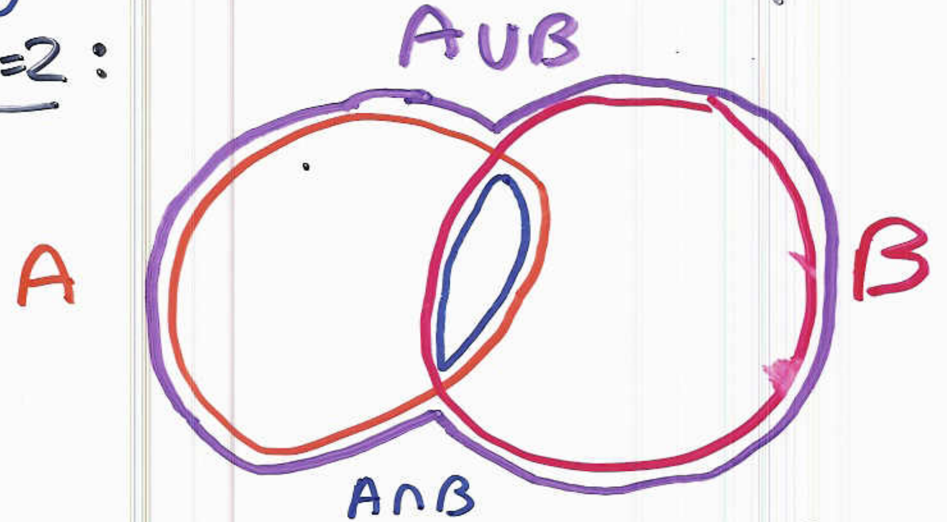
cracks ← excursion set A_x..

PROOF: Hadwiger (1959): ψ set functional

- $\psi(A)$ invariant under rotation, translation of A
- additive: $\psi(A \cup B) = \psi(A) + \psi(B) - \psi(A \cap B)$

e.g. intrinsic volumes $\psi(A) = \mu_d(A)$:

D=2:



$\mu_0(A) = EC(A)$: $EC(A \cup B) = EC(A) + EC(B) - EC(A \cap B)$ ✓

$\mu_1(A) = \frac{1}{2}$ perimeter length of A ✓

$\mu_2(A) =$ surface area of A ✓

Converse is also true: intrinsic volumes are the only functionals that obey additivity:

$$\boxed{\psi(A) = \sum_{d=0}^{D} \mu_d(A)\, c_d}$$ ← CONSTANTS

PROOF: $\psi(S) = E(EC(S \cap A_x))$

- invariant? ✓ since $X(\underset{\sim}{t})$ isotropic
- additive? ✓ since EC additive

$$\psi(S) = E(EC(S \cap A_x)) = \sum_{d=0}^{D} \mu_d(S) \cdot \rho_d(x)$$

THE GLOBE AND MAIL
SATURDAY, JANUARY 29, 2005

Cosmic oddity casts doubt on theory of universe

BY DAN FALK

A new analysis of the "echo" of the Big Bang has left cosmologists scratching their heads and could throw a monkey wrench into efforts to understand how the universe began.

U.S. and European scientists analyzed the distribution of "hot" and "cold" regions — areas that are putting out greater or less amounts of energy than the average — of the cosmic microwave background radiation (the so-called echo). What they found was unexpected: an apparent correlation between those hot and cold spots and the orientation and motion of our solar system.

"All of this is mysterious," says Glenn Starkman, a Canadian physicist based at Case Western Reserve University in Cleveland and one of the authors of a recent paper in Physical Review Letters that outlined the finding. "And the strange thing is, the more you delve into it, the more mysteries you find."

The study, by Case Western scientists and the European Centre for Nuclear Research in Geneva, is based on data from the WMAP satellite, the NASA spacecraft that began mapping the cosmic microwave background (CMB) radiation in fine detail in 2001.

The observed correlation is troubling on several fronts.

First of all, there is no reason to believe that the finding reflects any physical connection between our local astronomical neighbourhood and the universe at large.

As Dr. Starkman puts it: "None of us believe that the universe knows about the solar system, or that the solar system knows about the universe."

Far more plausible, he says, is that something within our solar system is producing or absorbing microwaves. That means that anyone doing cosmology would have to take into account such "local" contamination.

(The correlation involves the largest-scale fluctuations of the CMB radiation. If some of those fluctuations are a local rather than a cosmological phenomenon, it would mean that the truly cosmological large-scale fluctuations are even less intense than previously thought.)

There is, however, another possibility: The patterns seen by Dr. Starkman and his colleagues might simply be a fluke — an accidental alignment between the solar system and patterns in the CMB radiation.

If the correlation is real, however, it could cast doubt on the popular "inflation" model of the early universe. That model, which builds on the well-established Big Bang theory, says the universe underwent a period of incredibly rapid, exponential growth in the first split-second of its existence.

One of its predictions is that the universe should be nearly perfectly "smooth," that the CMB fluctuations should be equally intense at all scales.

An analogy with a musical instrument can be helpful: If you hit a drum, you hear many tones at the same time — a primary tone as well as many overtones, or "harmonics." The inflation model predicts that all the overtones in the CMB should be equally intense, but instead "we're missing the bass," Dr. Starkman says. "And what bass there is seems to be not generated by the universe, but by something local."

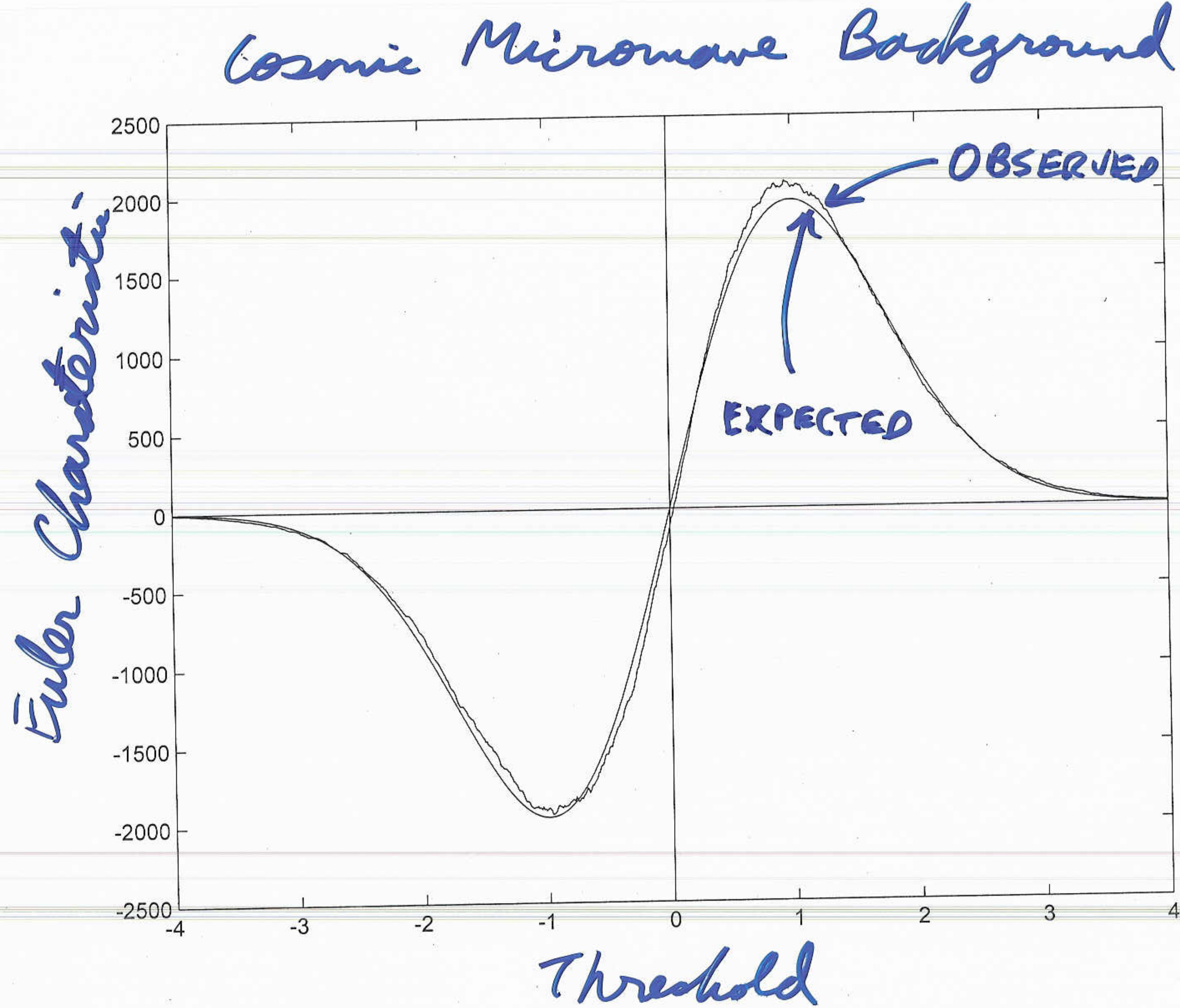
Other physicists are responding with caution to the finding.

"There is no way to judge the real significance of such a result," says Charles Bennett of NASA's Goddard Space Flight Center in Greenbelt, Md., the leader of the WMAP team.

It all depends on how we perceive "chance," and how we evaluate probabilities, Dr. Bennett says. The alignments seen in the CMB may seem unlikely, he says, but that doesn't necessarily mean that they require new physics to explain them.

He points out that "improbable things happen frequently because there are lots of opportunities for them to occur." In other words, he says, the newly discovered CMB correlations are most likely the product of chance.

Dan Falk is a science journalist based in Toronto.



Cosmic Microwave Background
Euler Characteristic
2500
2000
1500
1000
500
0
-500
-1000
-1500
-2000
-2500
-4
-3
-2
-1
0
1
2
3
4
OBSERVED
EXPECTED
Threshold

Back to P-value: (Adler, 2000)

$$\left| P\left(\max_{\underline{t}\in S} X(\underline{t}) \geq x\right) - E\big(EC(S\cap A_x)\big)\right| = O\left(e^{-\frac{1}{2}x^2\alpha}\right)$$

$$\boxed{\alpha > 1}$$

0

$$\left(\text{Poly. degree } D\text{-}1 \text{ in } x\right) e^{-\frac{1}{2}x^2} + \int_x^\infty \frac{e^{-\frac{1}{2}u^2}}{\sqrt{2\pi}}\,du$$

error $\rightarrow 0$ exponentially faster than last term

$\Rightarrow E(EC(S\cap A_x))$ gives first $D+1$ terms!

Taylor, Takemura, Adler (2005):

S convex, spatial correlation $h(\underline{t})$ monotone,

$$\boxed{\alpha = \frac{1}{1-\mathrm{Cor}(X,\ddot{X})^2}}$$

D=1:

e.g. $X(t) = Z_1 \sin(t) + Z_2 \cos(t)$

$\sim N(0,1)$ $\sim$

$\ddot{X} = -X$, $\alpha = \infty$, $E(EC(S\cap A_x))$ is exact!

- $n_1 = 163$ normal adult males
  $n_2 = 158$ " " females

  $n = 321$

- Y = cortical thickness, mm.

- X = T statistic (2-sample) $= \dfrac{\bar{Y}_1 - \bar{Y}_2}{s.e.(\bar{Y}_1 - \bar{Y}_2)} \sim t_{319\,d.f.}$

- Random Field Theory: $P(T > 4.4) \simeq 0.05$
  (corrected for search)

Non-isotropic random fields (W'98, Taylor + Adler, 2003)

e.g. $\underset{\sim}{t} \in$ manifold such as cortical surface

Idea: (Sampson + Guttorp, '92):

Replace Euclidean metric by variogram:

$$dist(\underset{\sim}{t}_1, \underset{\sim}{t}_2) = Var(X(\underset{\sim}{t}_1) - X(\underset{\sim}{t}_2))^{\frac{1}{2}}$$

Result:

$$E(EC(S \cap A_x)) = \sum_{d=0}^{D} \mathcal{L}_d(S, \underset{D \times D}{\Lambda(\underset{\sim}{t})})\, \rho_d(x)$$

"Lipschitz-Killing curvature"

$Var(\dot{X}(\underset{\sim}{t}))$

for isotropic GRF, $Var(\dot{X}) = I$

- transfers smoothness information from EC densities to Lip.-Kill. Curv.
- BUT very tricky to evaluate LKC in general

Isotropic case: $\mathcal{L}_d(S, \underset{D \times D}{I}) = \mu_d(S)$ (as before)

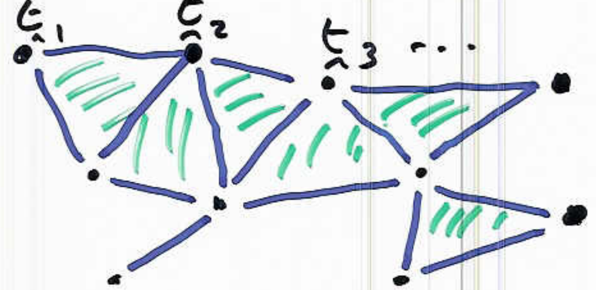
$$\mathcal{L}_d(S, \underset{D \times D}{I} \lambda^2) = \mathcal{L}_d(\lambda S, I) = \mu_d(\lambda S) = \lambda^d \mu_d(S)$$

$\{\lambda \underset{\sim}{t} : \underset{\sim}{t} \in S\}$

Really need an estimator of L.K.C.:

S = simplicial complex

eg $D=2$:

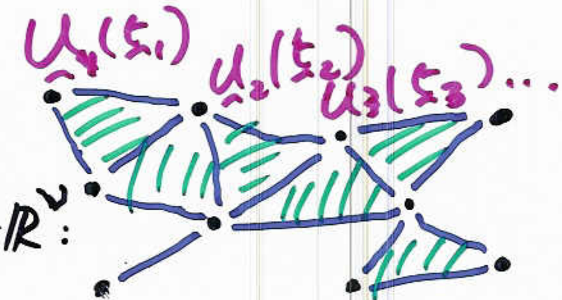


Suppose $\underset{\sim}{X}(\underline{t}) = \begin{pmatrix} X_1(\underline{t}) \\ \vdots \\ X_\nu(\underline{t}) \end{pmatrix}$ i.i.d. $X(\underset{\sim}{t})$ under H_0

e.g. ~ residuals from a linear model.

Let $$\underset{\sim}{u}(\underline{t}) = \frac{\underset{\sim}{X}(\underline{t})}{\|\underset{\sim}{X}(\underline{t})\|}, \quad \|\underset{\sim}{u}(\underline{t})\| = 1$$

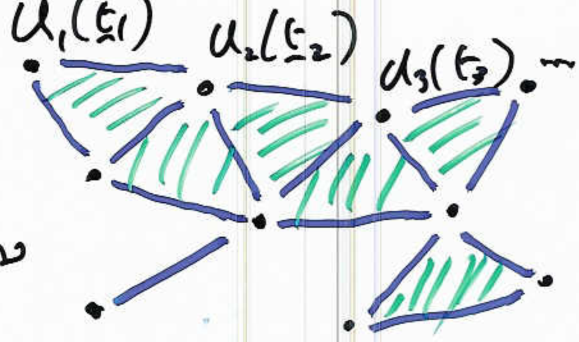
Define $\hat{S}$ = "isotropic" S by replacing coord $\underline{t} \in \mathbb{R}^D$ by $\underset{\sim}{u}(\underline{t}) \in \mathbb{R}^\nu$:



Then: $$\boxed{\hat{\mathcal{L}}_d(S, \Lambda(\underline{t})) = \mu_d(\hat{S})}$$

As mesh size $\to 0$,
- M.L.E
- unbiased

How do we calculate $\mu_d(\hat{S})$?

$\hat{S} =$ 

$\subset \mathbb{R}^D$

Not smooth .... could smooth slightly ... mess!

However we know $\mu_d(\text{simplices})$

e.g. $D=3$

$$\mu_0(\text{tetrahedron}) = 1$$

$$\mu_1(\text{tet}) = \sum_{\text{edges}} \frac{(\pi - \text{interior angle})}{2\pi} \times \begin{pmatrix}\text{edge}\\ \text{length}\end{pmatrix}$$

$$\mu_2(\text{tet}) = \tfrac{1}{2}\text{ surface area}$$

$$\mu_3(\text{tet}) = \text{volume}$$

and:

$$\hat{S} = \bigcup \text{simplices}$$

and:

$$\mu_d(S_1 \cup S_2) = \mu_d(S_1) + \mu_d(S_2) - \mu_d(S_1 \cap S_2)$$

~~so~~ we should be able to do it, but it is still a $\boxed{\text{mess}}$: 40962 Δ's and $\frac{40962 \times 40961}{2}$ intersections.....

Theorem (Taylor + W'2005)

D=3 case: $\mu_3(\hat{S}) = \sum_{tet} \mu_3(tet)$

$$\mu_2(\hat{S}) = \sum_{tri} \mu_2(tri) - \sum_{tet} \mu_2(tet)$$

$$\mu_1(\hat{S}) = \sum_{edges} \mu_1(edge) - \sum_{tri} \mu_1(tri) + \sum_{tet} \mu_1(tet)$$

$$\mu_0(\hat{S}) = \sum_{points} \underbrace{\mu_0(points)}_{=1} - \sum_{edge} \underbrace{\mu_0(edge)}_{=1} + \sum_{tri} \underbrace{\mu_0(tri)}_{=1} - \sum_{tet} \underbrace{\mu_0(tet)}_{=1}$$

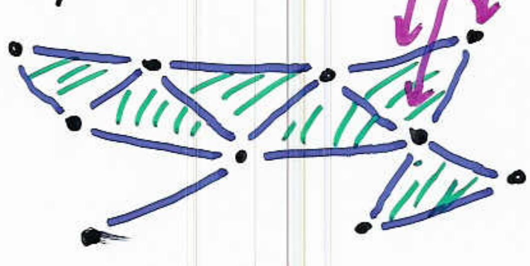
$$= \#points - \#edges + \#tri - \#tet = EC!$$

Proof Sommerville's Identity from quantum gravity (!).

Heuristic: Write $\hat{S}$ as a union of disjoint interiors:

interior

$$\hat{S} = \bigcup_{tet} tet^{\circ} \cup \bigcup_{tri} tri^{\circ} \cup \bigcup_{edge} edge^{\circ} \cup \bigcup_{points} points$$



then $\mu_d(A^{\circ}) = \mu_d(A) - \mu_d(\partial A)$

$$= (-1)^{\dim(A)-d}\, \mu_d(A)$$

not really defined....

Scale space intrinsic volumes

$X(\underset{\sim}{t}, w)$ not isotropic in $S \times [w_1, w_2]$

Taylor, Adler (2003): replace intrinsic volumes by Lipschitz-Killing curvatures:

$$\mathcal{L}_d(S \times [w_1, w_2]) = \frac{w_1^{-1} + w_2^{-1}}{2} \mu_d(S) +$$

$$\sum_{j=0}^{\lfloor (D-d+1)/2 \rfloor} \frac{w_1^{-d-2j+1} - w_2^{-d-2j+1}}{d+2j-1} \times \frac{(-1)^j (d+2j-1)!}{(1-2j)(4\pi)^j (d-1)!} \mu_{d+2j-1}(S) \times K^{\frac{1-2j}{2}}$$

where $K = \int \left(\underset{\sim}{t}' \dot{\underset{\sim}{f}} + \frac{D}{2} f \right)^2 d\underset{\sim}{t} \Big/ \int f^{\,2} d\underset{\sim}{t}$

$= D$ for Gaussian and Marr wavelet

Confidence region for signal location

If signal $\propto$ spatial correlation function

$$-2 \log(\text{likelihood}) = X(t)^2 + \text{const.}$$

$\sim N(\, , \sigma^2)$

Usual theory works:

Approx. (asymptotic) $100(1-\alpha)\%$ confidence region:

$$C = \{ \underset{\sim}{t} : X_{max}^2 - X(t)^2 \leq \chi^2_{D,\alpha} \}$$

$$= \{ \underset{\sim}{t} : X(\underset{\sim}{t}) \geq \sqrt{X_{max}^2 - \chi^2_{D,\alpha}} \}$$

$D=3, \alpha=0{\cdot}05, \; 7{\cdot}89$

i.e. find local max, X_{max}^2, drop down $\chi^2_{D,\alpha}$, then **threshold**.

SUMMARY: When can you use this?

- Repeated co-registered images
- Interested in localised differences/activations - most of image is not affected.
- Thresholding is optimal for detection (perhaps after smoothing)
- Smooth (not necessarily equally so)
- If not smooth, good old Bonferroni is often better!
- Alternative approach altogether: False Discovery Rate ....